Improvements to a Branch-Cut-and-Price Algorithm for the Exact Solution of Parallel Machines Scheduling Problems

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Outline

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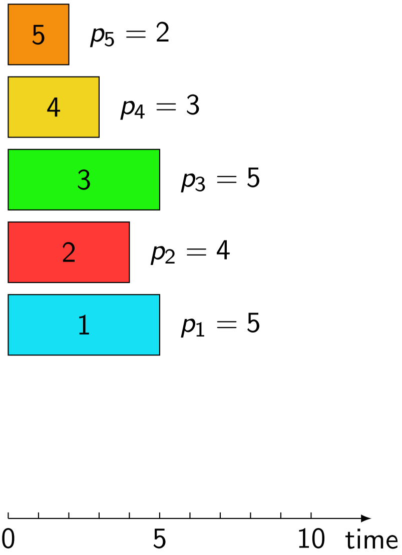

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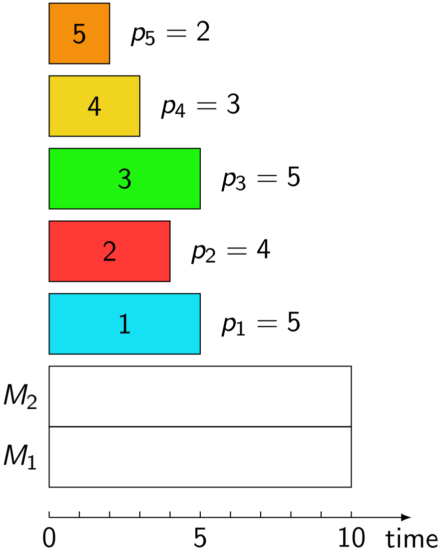

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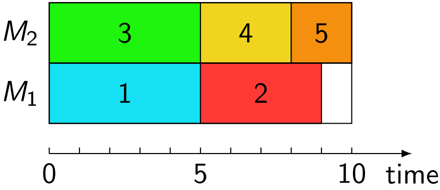

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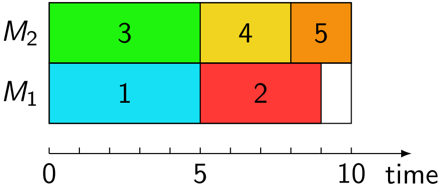
Weighted Tardiness:

- Due dates d_j
- Weights w_j
- Minimize $\sum w_j T_j$



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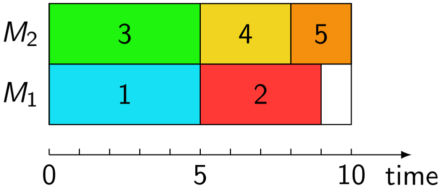

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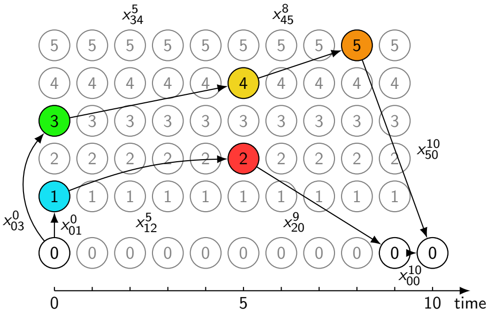

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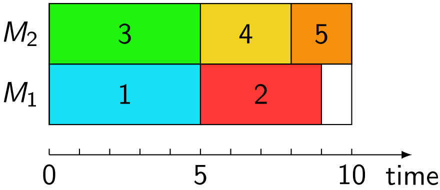

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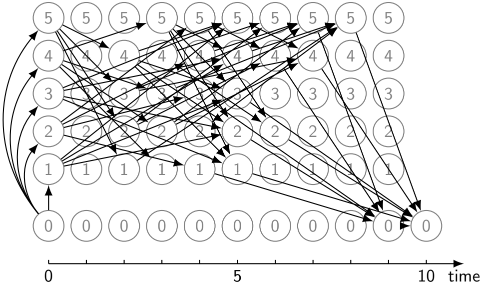

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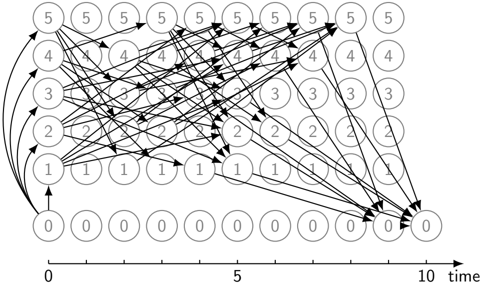

$$\min \sum_{i \in J_0} \sum_{j \in J \setminus \{i\}} \sum_{t=p_i}^{T-p_j} f_j\left(t+p_j\right) x_{ij}^t$$

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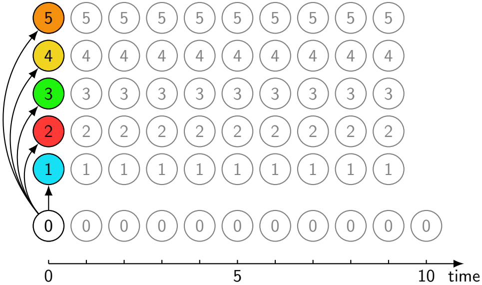

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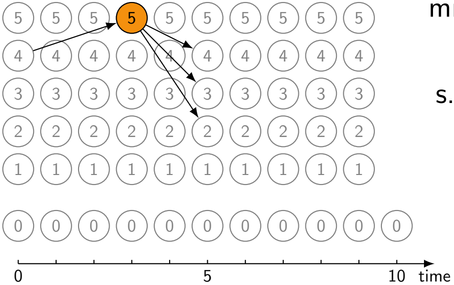

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$$\text{s.t.} \quad \sum_{\substack{j \in J_0 \setminus \{i\},\\ t-p_j \geq 0}} x_{ji}^t - \sum_{\substack{j \in J_0 \setminus \{i\},\\ t+p_i+p_j \leq T}} x_{ij}^{t+p_i} = 0$$

$$(\forall i \in J;\ t = 0, \ldots, T - p_i)$$

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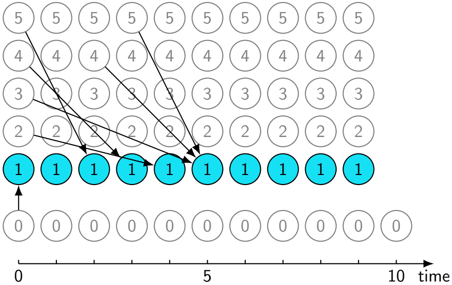

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$$\text{s.t.} \quad \sum_{i \in J_0 \setminus \{j\}} \sum_{t=p_i}^{T-p_j} x_{ij}^t = 1 \quad (j \in J)$$

$$x \in Z^+$$

ATIF Reformulation

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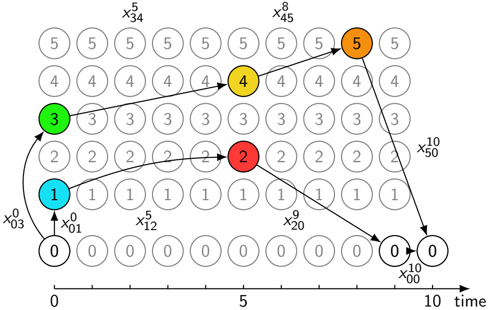

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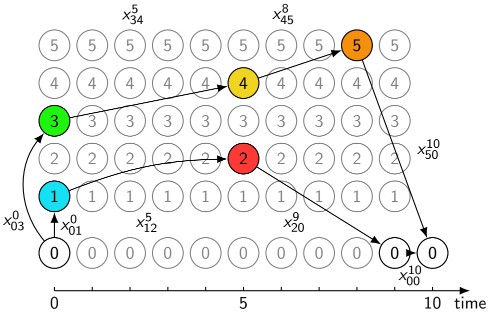

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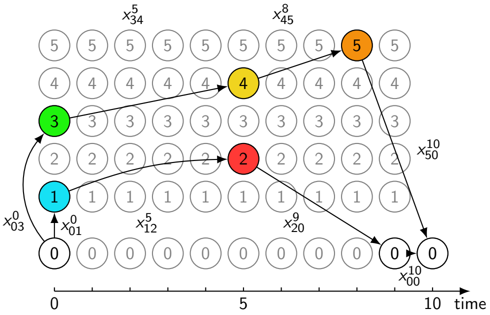

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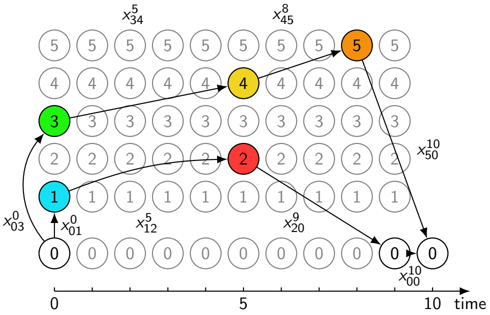

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- $x^t_{ij} = \sum_{p \in P} q^{tp}_{ij} \lambda_p$ (q^{tp}_{ij} constant)

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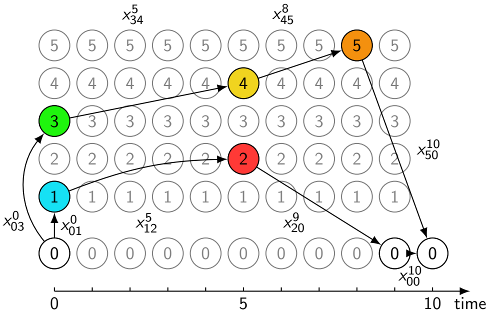

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- Substituting in the ATIF without flow conservation

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$$\lambda \geq 0$$

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$$\sum_{a^t \in A} \alpha_{al}^t x_a^t \qquad \geq b_l \qquad (\forall\, l \in \{n+1, \ldots, r\})$$

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- The cuts are robust: Pricing subproblem (shortest path) not changed

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- After Root, if $|A| \leq 200.000$: Feed reduced ATIF to MIP Solver (CPLEX 11.1)

1. The Parallel Machines Scheduling Problem
2. The BCP of Pessoa, Uchoa, Poggi and Rodrigues (2010)
3. The Improved Algorithm
   - Newly Proposed Cuts over Extended Variables
   - Alternative Time-Indexed Formulations
   - New Cuts over Time-Indexed Formulations
4. Experiments
5. Conclusions

The Overload Elimination Cuts

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$$u^t = \sum_{a^t \in \delta^-(S)} x_a^t \quad (t = 1, \ldots, T)$$

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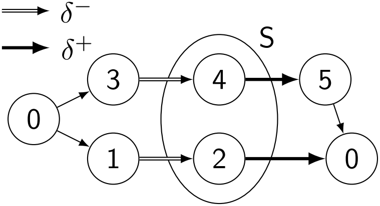

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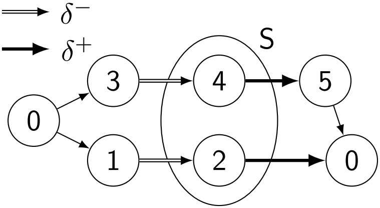

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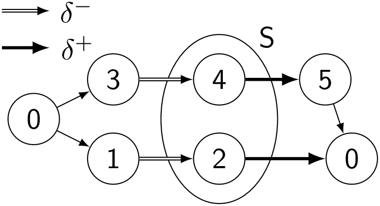

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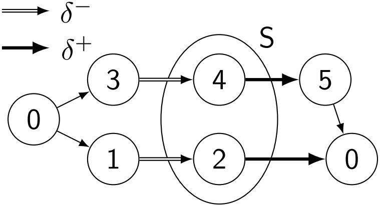

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- Separation: A specialized genetic algorithm

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\text{s.t.} \quad & \sum_{t=p_j}^{T} y_j^t = 1 && (j \in J) \\
& \sum_{j \in J} \sum_{t'=t}^{\min\{t+p_j-1,\, T\}} y_j^{t'} \leq m && (t = 1, \dots, T) \\
& y \in \{0, 1\}
\end{aligned}
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- 4 different time-indexed formulations (R_y, R_z, F_y, F_z)

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$\underline{R_y}$ R_z F_y F_z

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\min \quad & \sum_{j \in J} \sum_{t=p_j}^{T} f_j(t) \left(z_j^t - z_j^{t-1} \right) & \\
\text{s.t.} \quad & z_j^{p_j - 1} = 0 & (j \in J) \\
& z_j^{t-1} \leq z_j^t & (j \in J;\ t = p_j, \ldots, T) \\
& z_j^T = 1 & (j \in J) \\
& \sum_{j \in J} \left(z_j^{\min\{t+p_j-1,\, T\}} - z_j^{t-1} \right) \leq m & (t = 1, \ldots, T) \\
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\text{s.t.} \quad & \sum_{t=p_j}^{T} y_j^t = 1 && (j \in J) \\
& \sum_{j \in J} y_j^{p_j} = m \\
& \sum_{j \in J | t \geq p_j} y_j^t \geq \sum_{j \in J} y_j^{t+p_j} && (t = 1, \ldots, T) \\
& y \in \{0, 1\}
\end{aligned}
$$

Alternative Time-Indexed Formulations

R_y R_z F_y $\underline{F_z}$

$$\min \quad \sum_{j \in J} \sum_{t=p_j}^{T} f_j(t) \left(z_j^t - z_j^{t-1} \right)$$

$$\text{s.t.} \quad z_j^{p_j - 1} = 0 \qquad (j \in J)$$

$$z_j^{t-1} \leq z_j^t \qquad (j \in J;\ t = p_j, \ldots, T)$$

$$z_j^T = 1 \qquad (j \in J)$$

$$\sum_{j \in J} \left(z_j^{p_j} - z_j^{p_j - 1} \right) = m$$

$$\sum_{j \in J | t \geq p_j} \left(z_j^t - z_j^{t-1} \right) \geq \sum_{j \in J} \left(z_j^{t+p_j} - z_j^{t+p_j-1} \right) \qquad (t = 1, \ldots, T)$$

$$z \in \{0, 1\}$$

TIF Cuts by Projecting the ATIF Polytope

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$$\sum_{i \in J \setminus \{j\}} y_i^t \geq y_j^{t+p_j} \qquad (j \in J;\ t = 1, \ldots, T)$$

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- Separation: Solving a Minimum Cut Problem in a directed graph

How much of the Integrality Gap is closed?

How much of the Integrality Gap is closed?

Table: Root relaxation and cut separation results

		BCP-PMWT		BCP-PMWT-OTI	
n	m	Avg. Gap	Avg. Time	Avg. Gap	Avg. Time
40	2	0.525%	78.0	0.235%	51.9
40	4	0.456%	23.4	0.448%	18.8
50	2	0.379%	256.8	0.276%	193.8
50	4	0.571%	67.8	0.583%	29.9
100	2	0.878%	6297.0	0.114%	3398.8
100	4	0.494%	984.0	0.322%	481.6

Which is the best TIF?

Which is the best TIF?

Table: Comparison of Alternative Time-Indexed Formulations

	Average LP Time (s)				Average MIP Time (s)*				# Solved**			
n	Fy	Ry	Fz	Rz	Fy	Ry	Fz	Rz	Fy	Ry	Fz	Rz
40	0.72	0.84	7.17	0.97	63.17	351.97	122.92	58.28	12	10	12	12
50	1.77	1.98	47.08	2.43	53.46	150.26	70.56	16.47	13	11	14	16

*average times uses only instances solved with all 4 TIFs in up to 3,600 seconds
**we tested 12 instances of 40 jobs and 17 instances of 50 jobs

How much help is the Variable Fixation?

How much help is the Variable Fixation?

Table: Effect of Variable Fixation in the Rz Time-Index Formulation – Summary

	Average LP Time (s)		Average MIP Time (s)*		# Solved		Total
n	Fix.	w/ Fix.	Fix.	w/ Fix.	Fix.	w/ Fix.	
40	0.74	22.54	11.11	561.59	12	10	12
50	2.04	105.84	11.63	496.61	17	9	17

*average times uses only instances solved by both in up to 3,600 seconds

How much help are the Projected Cuts?

How much help are the Projected Cuts?

Table: Effect of Projected Cuts in the Rz Time-Indexed Formulation – Summary

		ATIF	TIF		
n	m	Root Gap	1st LP Gap	Root Gap	Gap Improv.
100	2	0.114%	0.294%	0.249%	16.76%
100	4	0.322%	0.660%	0.646%	11.20%

Overall Results

Overall Results

Table: Full Results - Summary

n	m	# Inst.	BCP-PMWT # Solved	BCP-PMWT Avg.* Time	BCP-PMWT-OTI # Solved	BCP-PMWT-OTI Avg.* Time
40		50	50	357.9	50	48.1
50		50	50	5734.9	50	241.9
100	2	25	18	22523.8	21	7058.5
100	4	25	16	37667.7	22	5672.0

*average times uses only instances solved by both

Branching vs Switching to MIP Solver

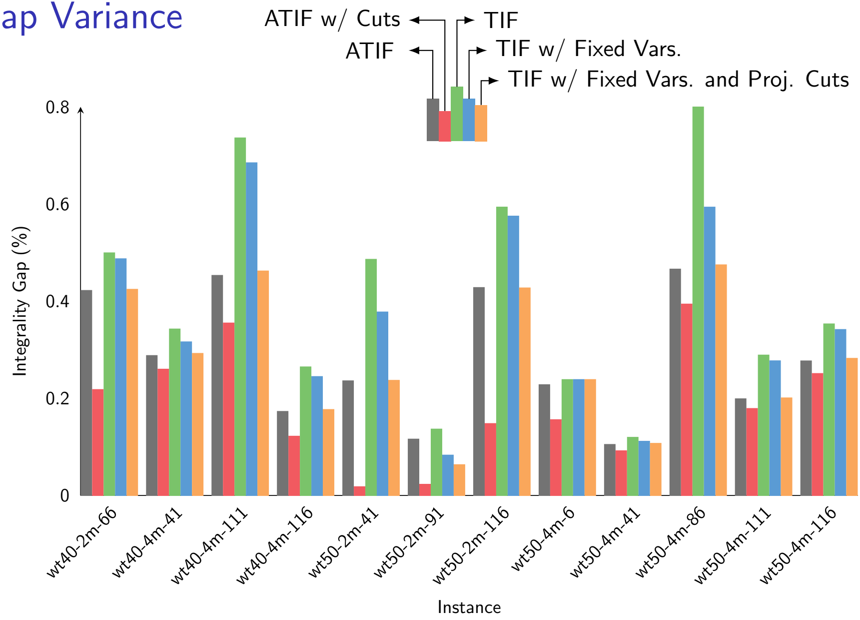
Branching vs Switching to MIP Solver

Table: BCP-PMWT-OTI Best Procedure

		BCP-PMWT				BCP-PMWT-OTI			
n	m	Root	BCP	ATIF MIP	Unsolved	Root	BCP	TIF MIP	Unsolved
40		38	2	10	0	38	1	11	0
50		33	4	13	0	33	3	14	0
100	2	13	2	3	7	16	1	4	4
100	4	7	5	4	9	7	1	14	3

Gap Variance

Gap Variance



Conclusions

Conclusions

- Results

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- Results
  - 9 instances solved for the first time

Conclusions

- Results
  - 9 instances solved for the first time
  - 84.1% running time decrease for other instances